Power Grid Analysis Based on a Macrocircuit Model

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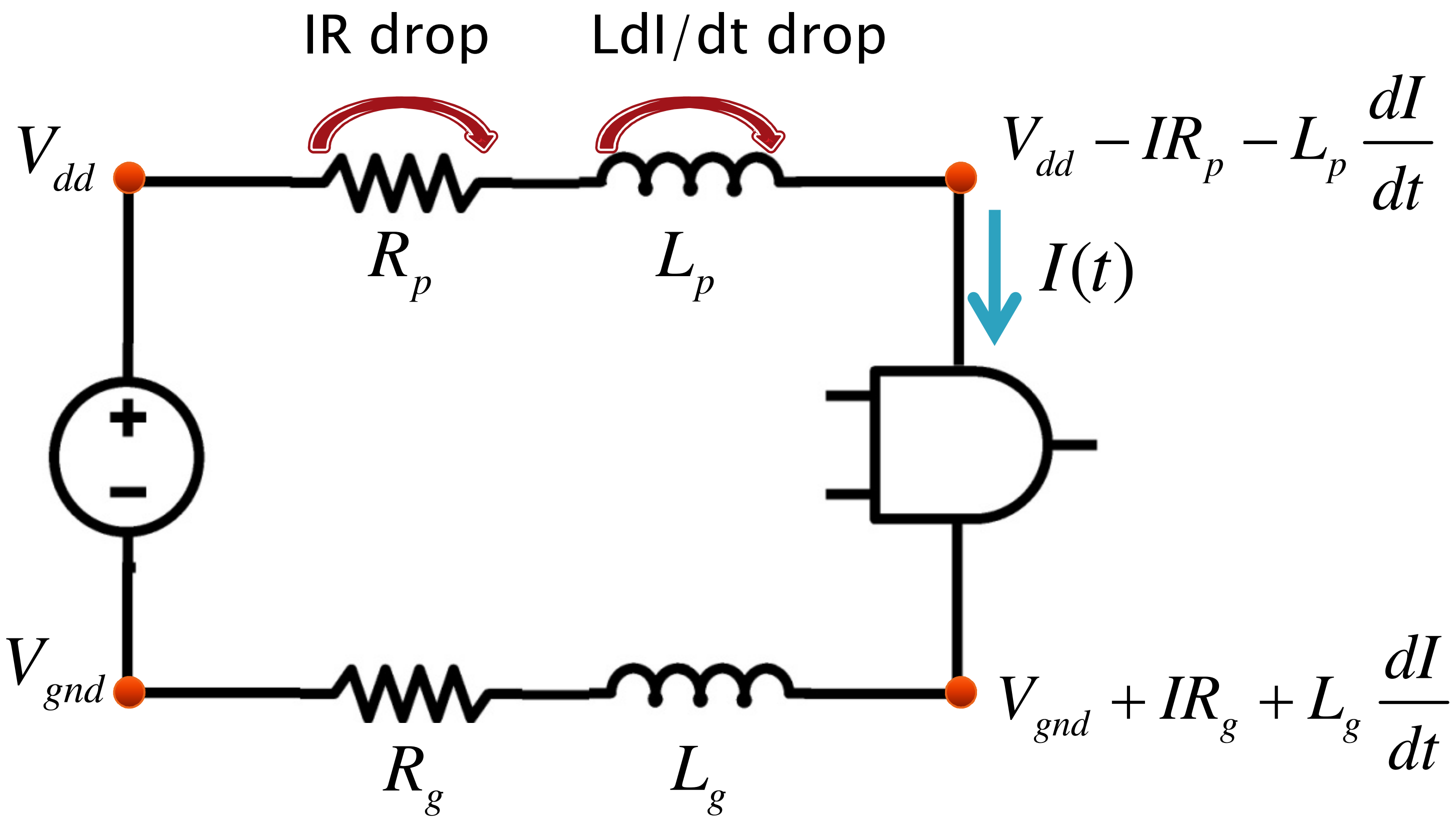
Outline

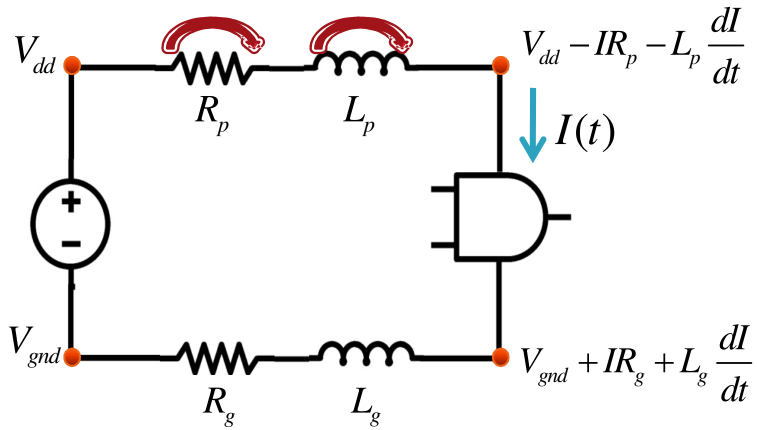
- **Review of Conventional Model**
- Basic Microcircuit Model
- Macrocircuit Model
- Feedback Between the Power Grid and Current Consumer
- Summary – Advantages and Potential Applications

Power Distribution Networks

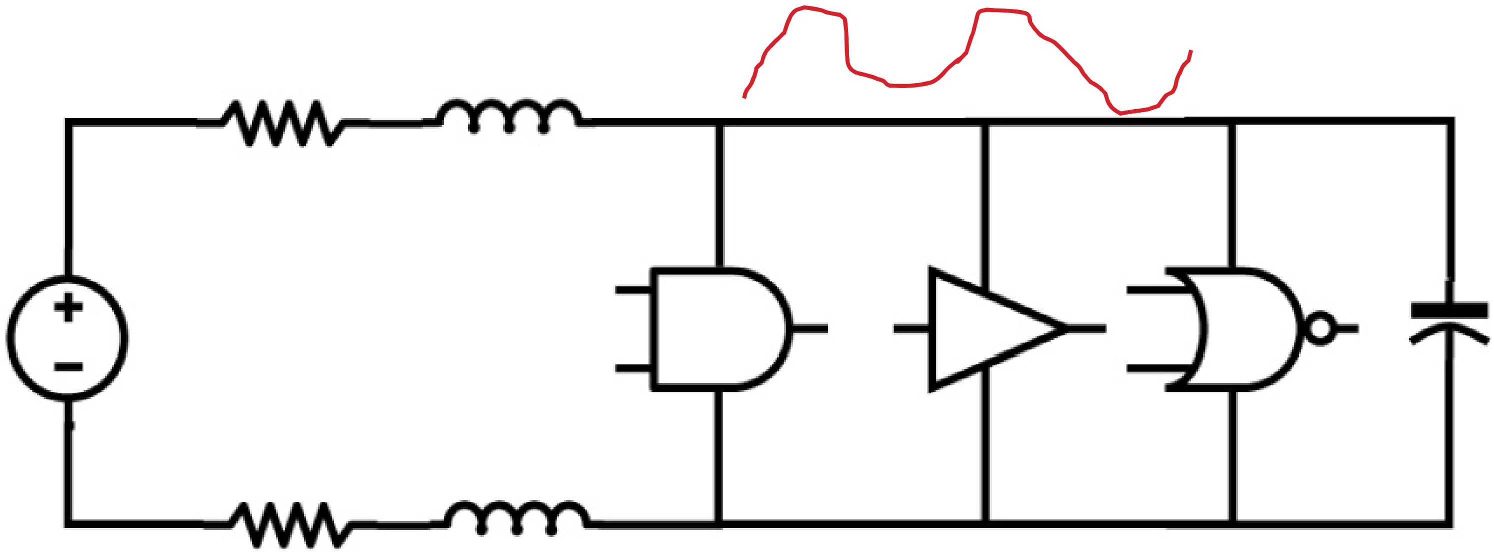


Power Grid Voltage Drop

- Slows the circuit – performance and functionality compromised
- Excessive voltage drop – logic errors
- Power grid design – **low voltage drop in all of the nodes**

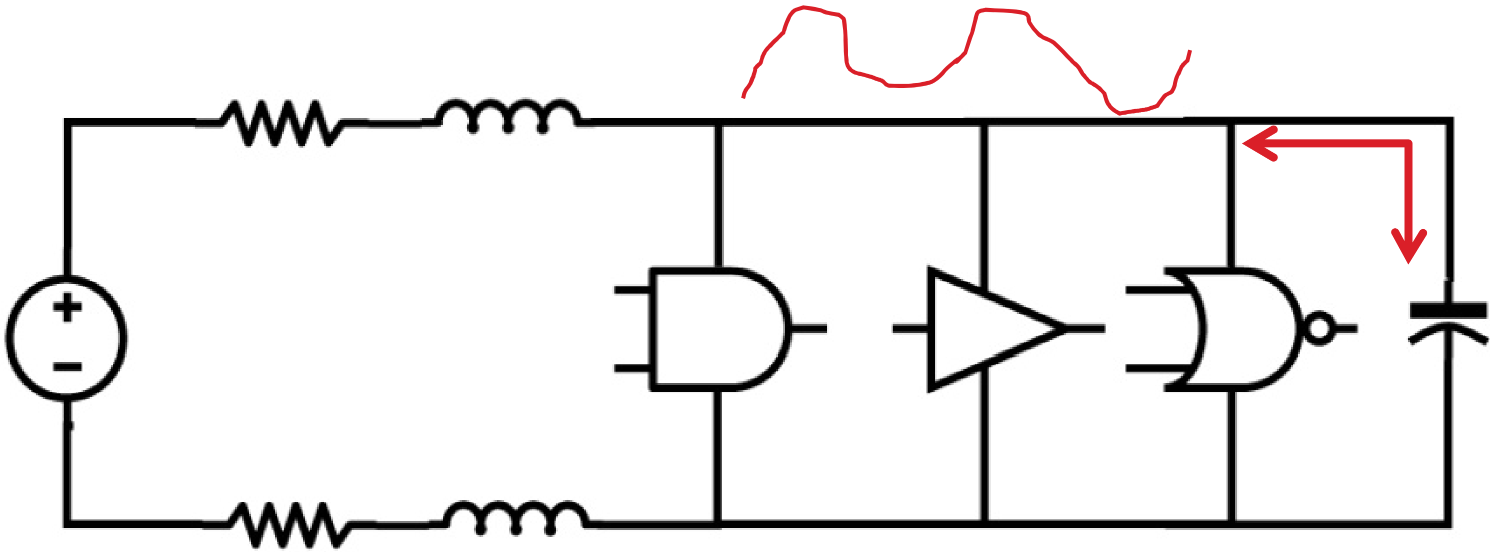


Decoupling Capacitors



Lowers the supply voltage fluctuations

Decoupling Capacitors



Lowers the supply voltage fluctuations

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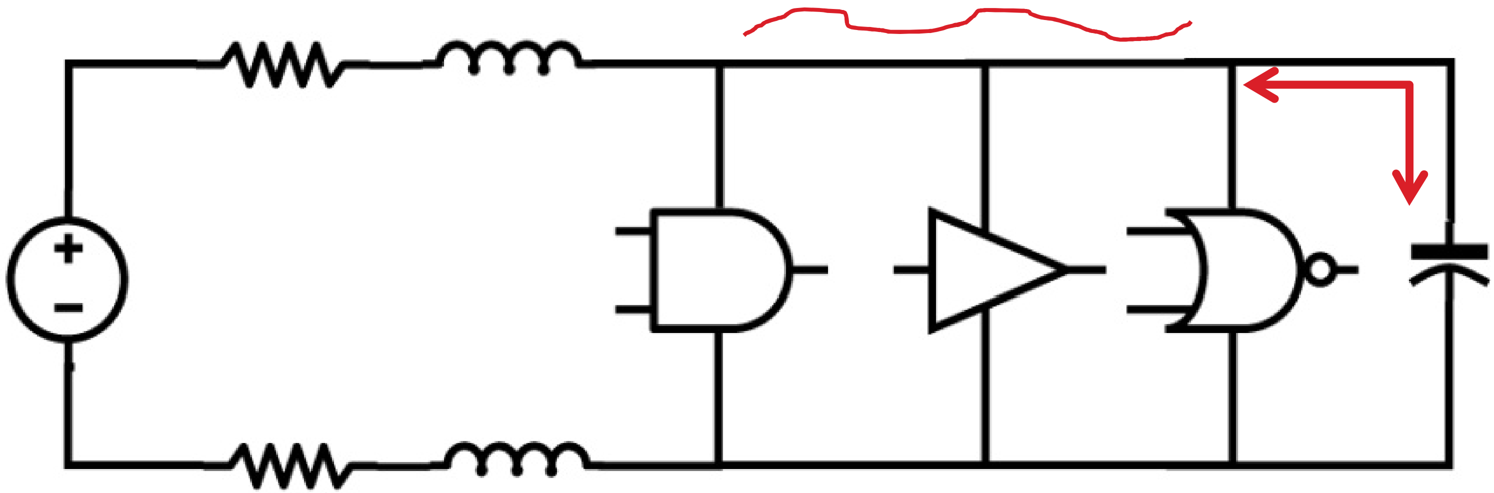
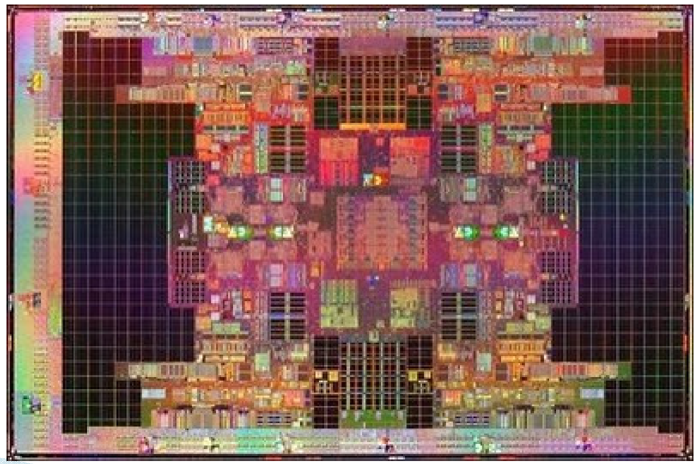
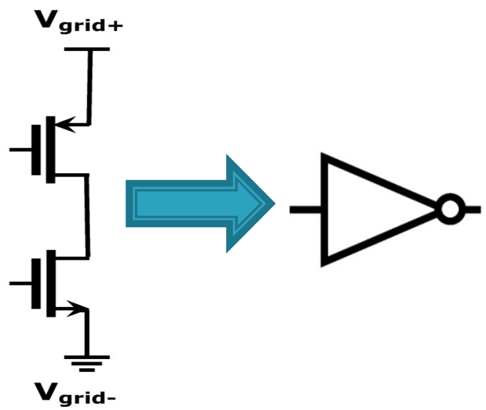


Lowers the supply voltage fluctuations

Current Consumers

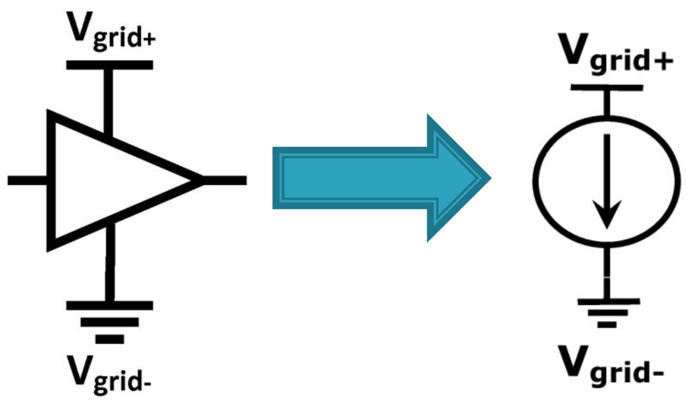
- Billions of transistors in a modern IC
- Every logic gate consumes current from the power supply
- Transistors are non-linear devices



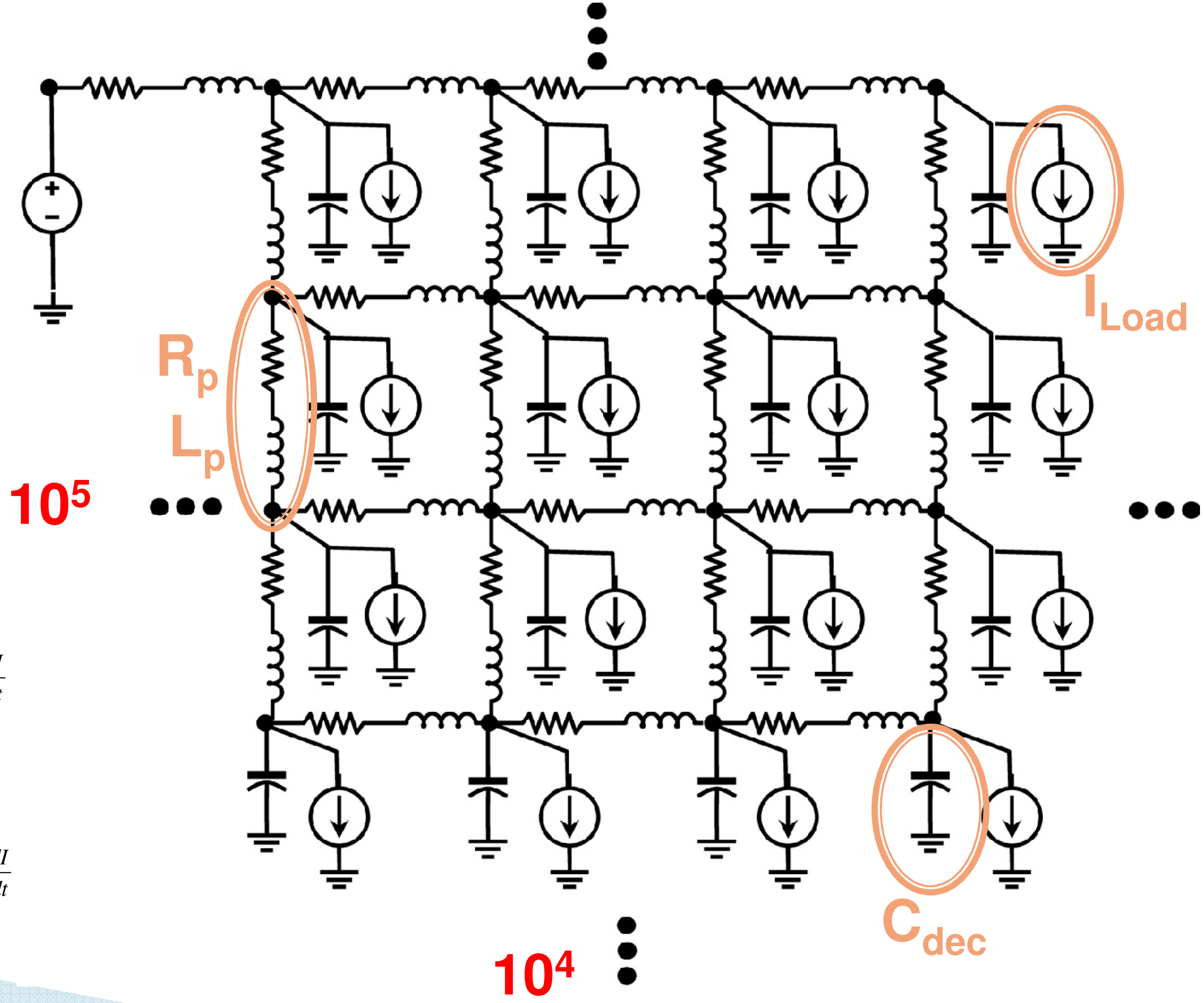


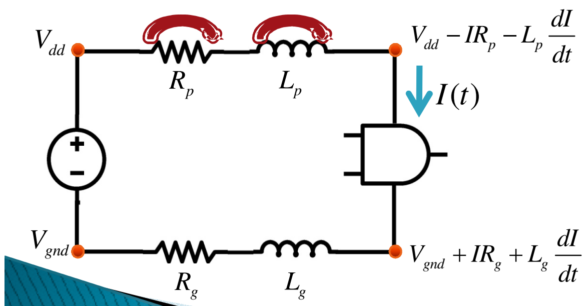
Conventional Current Consumer Model – Ideal Current Source

- Simulate individual circuit blocks, including transistors and parasitic elements within the power interconnect
- Replace each block by an ideal current source – a linear device



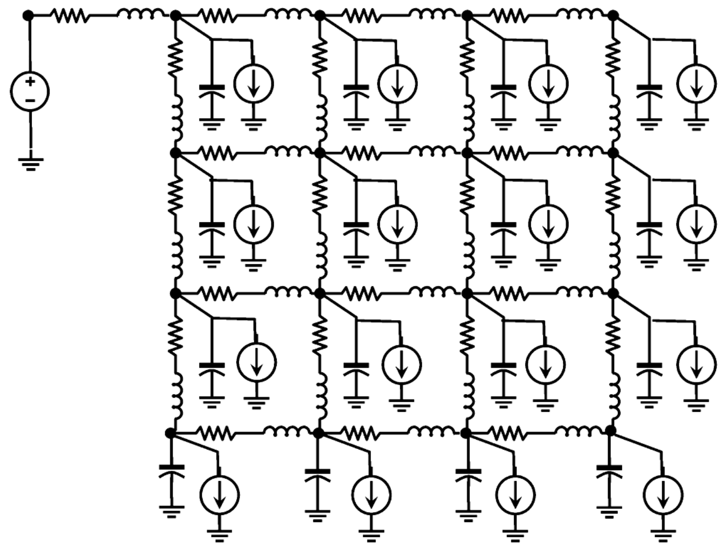
Conventional Power Grid Model





Conventional Current Consumer Model – Characterization

Old Model
Intuitive
Know every node's voltage
Significant computational time
Active supply as a consumer – misleading
Ignores effect of voltage drop on the current consumer
Complex



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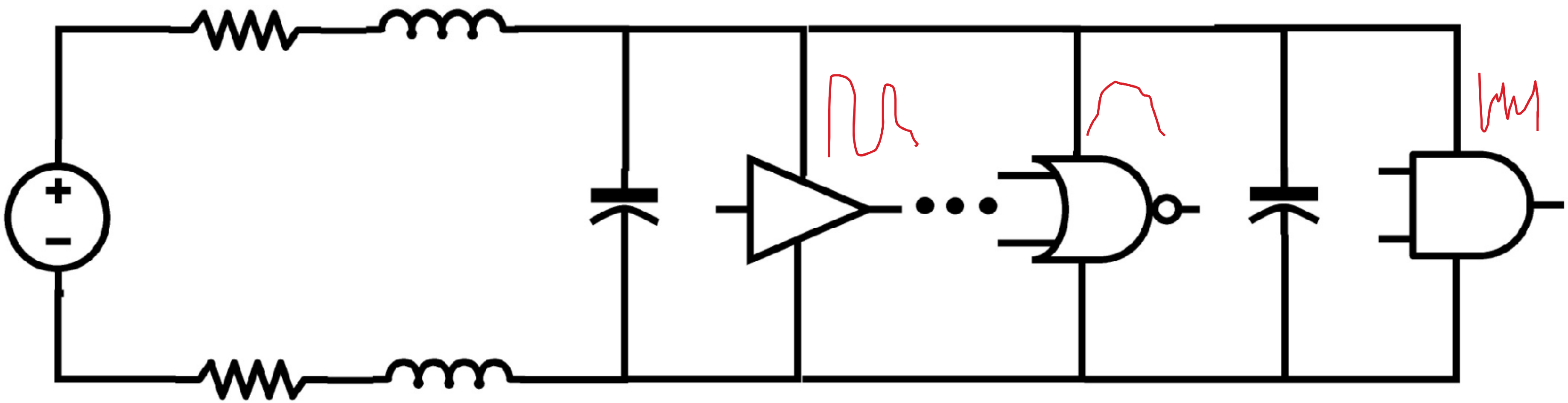
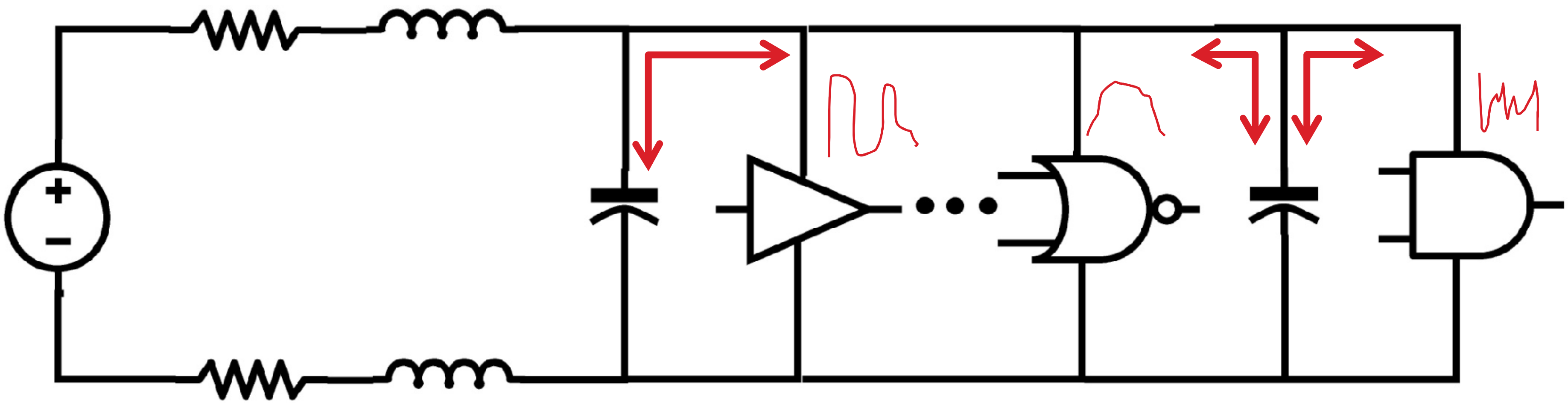
Power Supply Time Scale



Power Supply Time Scale
msec

Clock Time Scale
psec, nsec

Power Supply Time Scale



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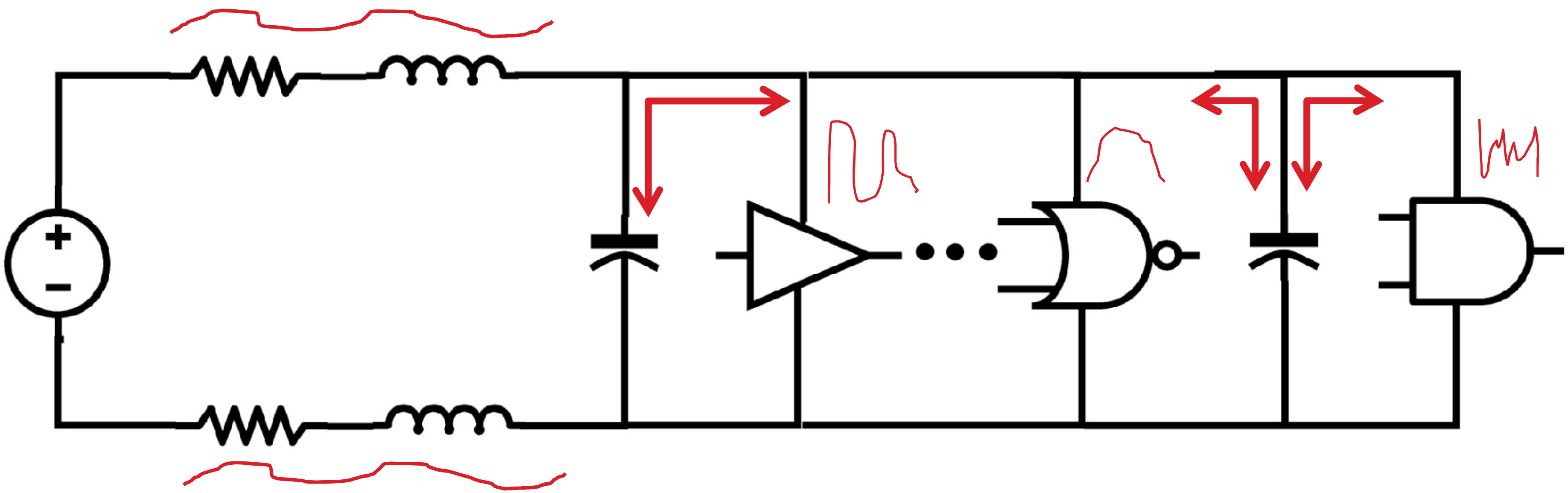

Power Supply Time Scale
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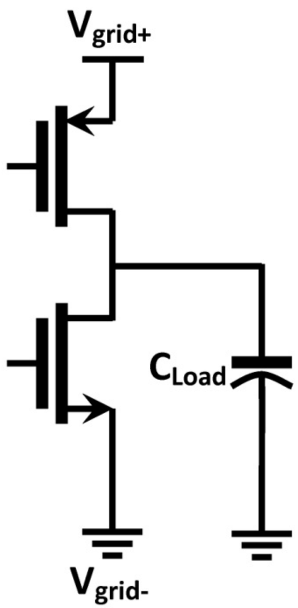
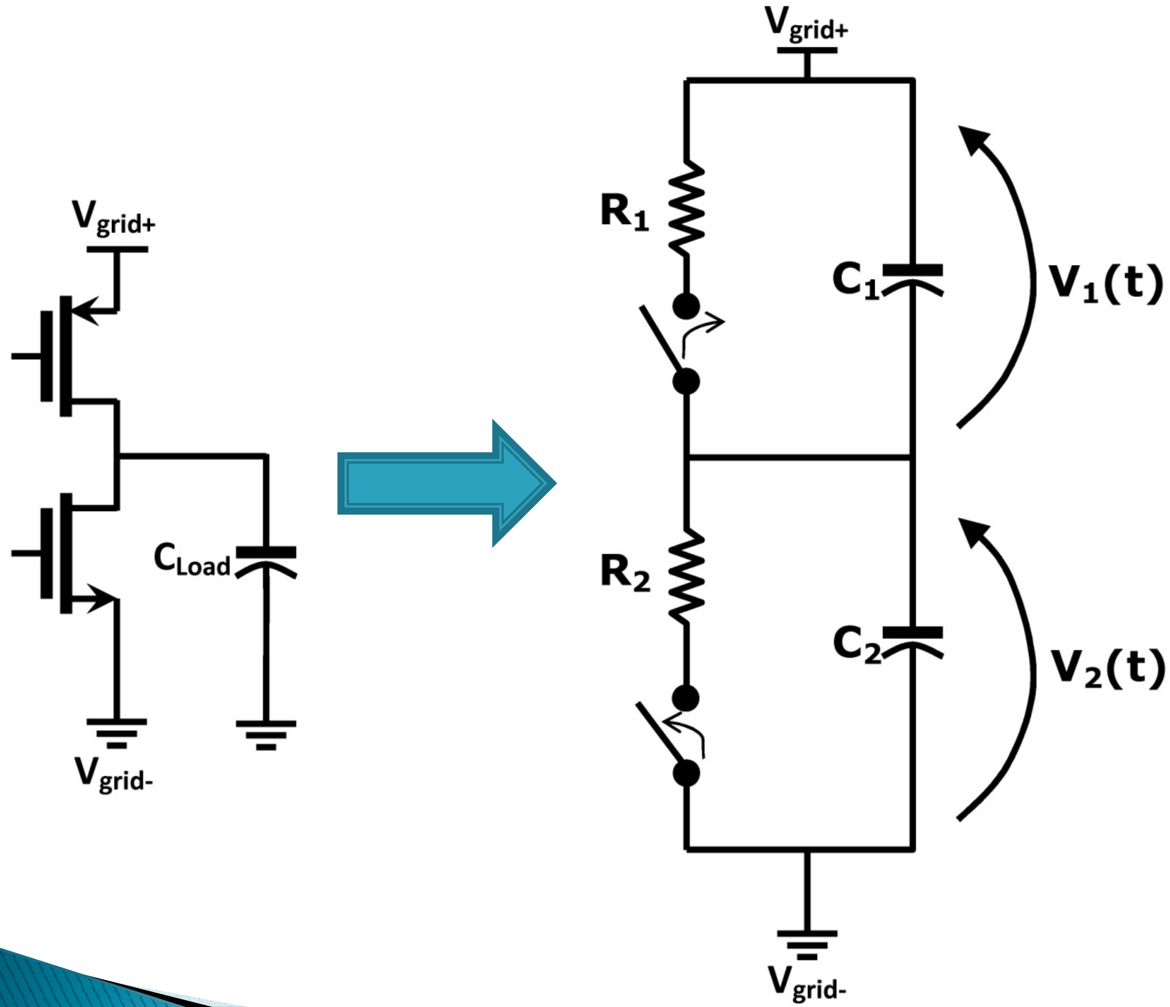
Microcircuit Model

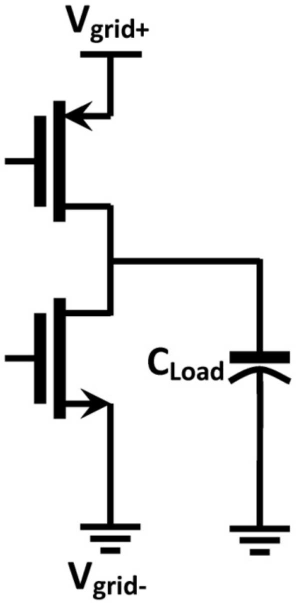


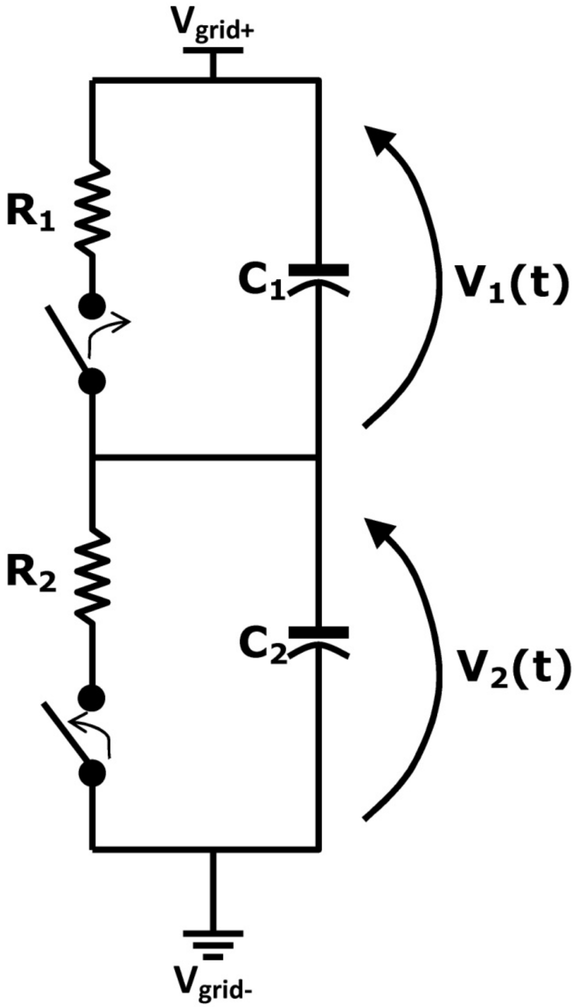
Microcircuit Model



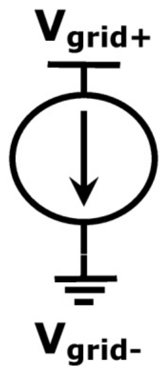
Clock Time Scale

Microcircuit Model



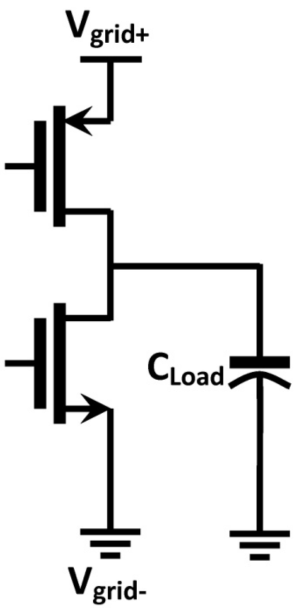


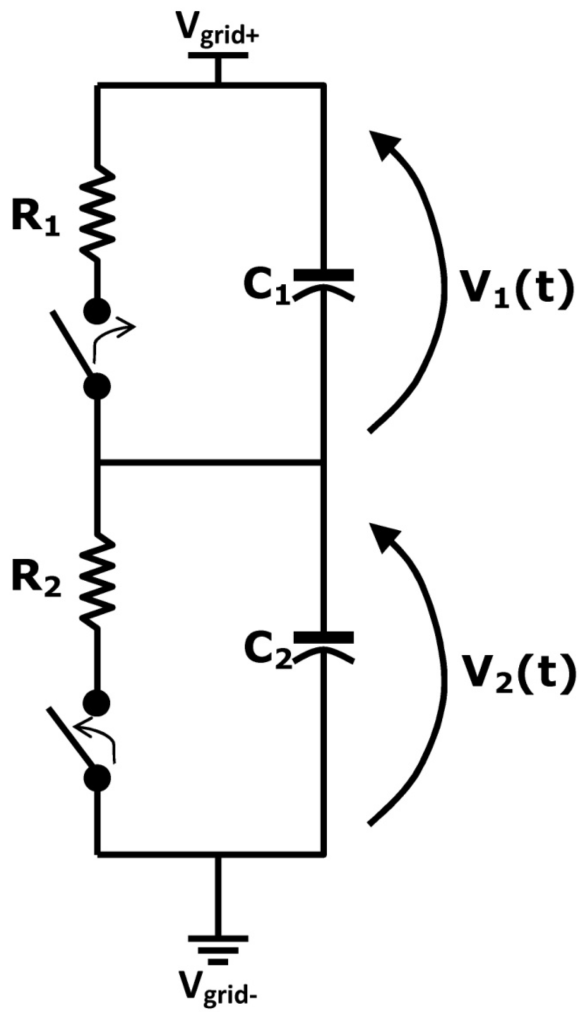
Clock Time Scale



Conventional Model

Microcircuit Model





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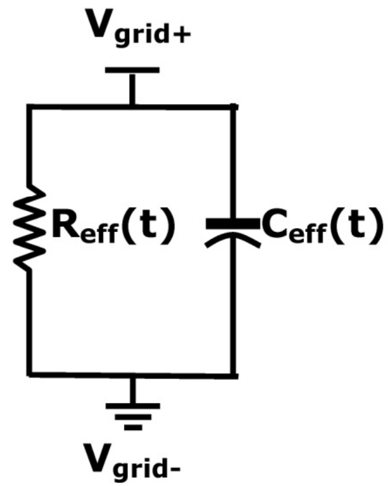
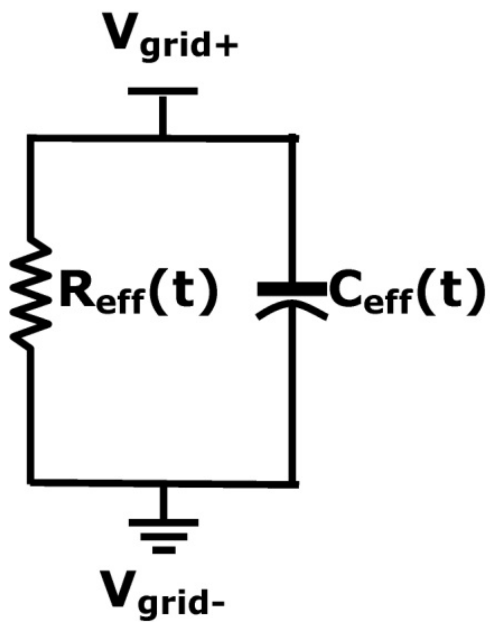


Power Supply Time Scale

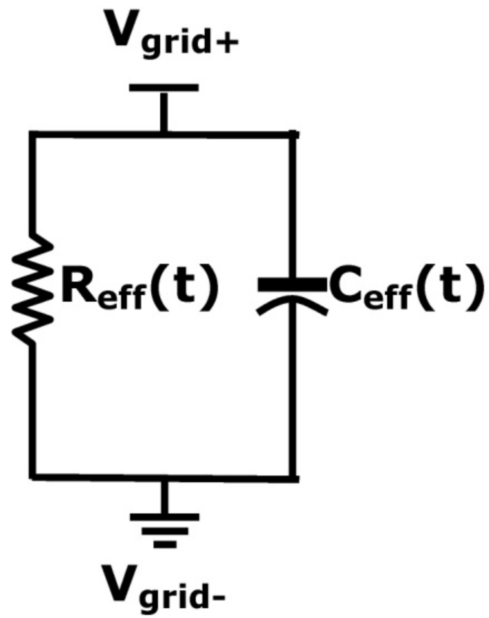
Effective Impedance

- Effective capacitance – accurately represents average energy stored in the element
- Effective resistance – accurately represents average power dissipated by the element

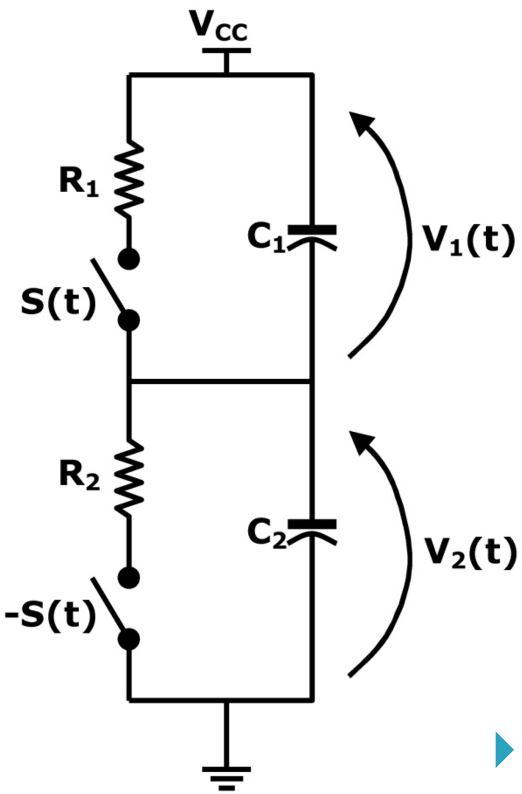


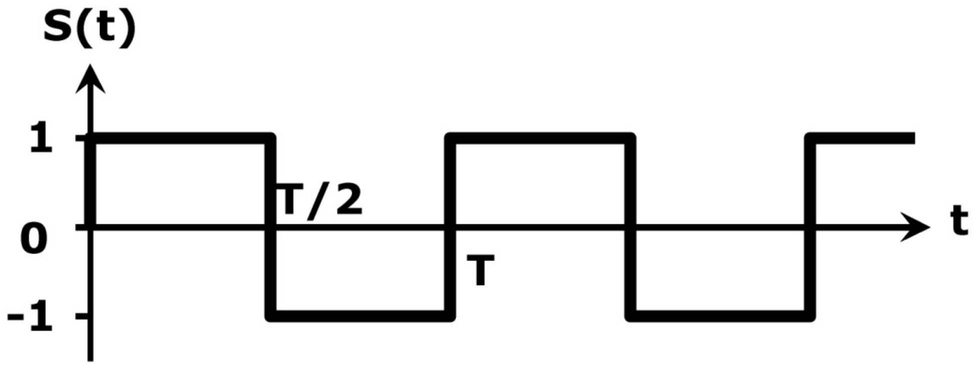
Determining the Effective Impedance

- Determine the effective impedance at the maximum switching rate of the element
- Determine the actual effective impedance of the element
  - A fraction of the maximum switching rate



Maximum Switching Rate



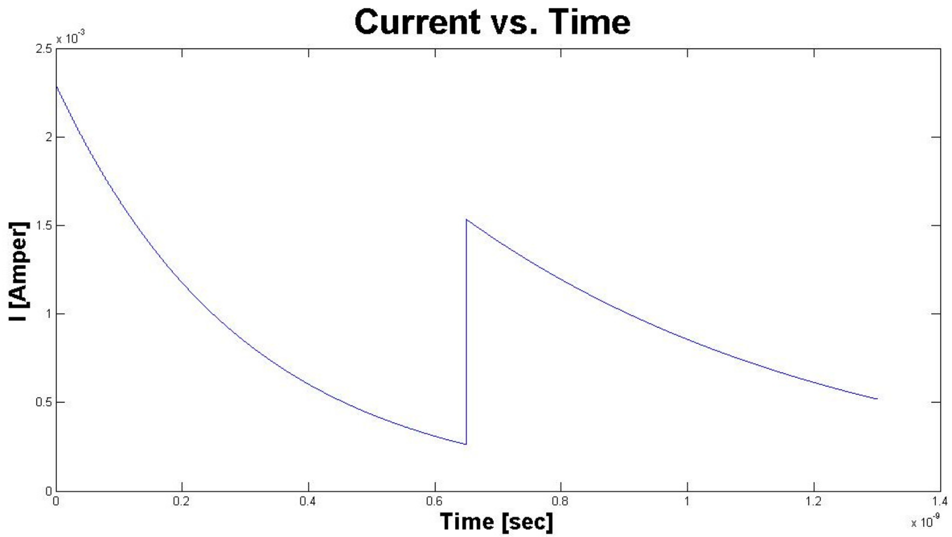


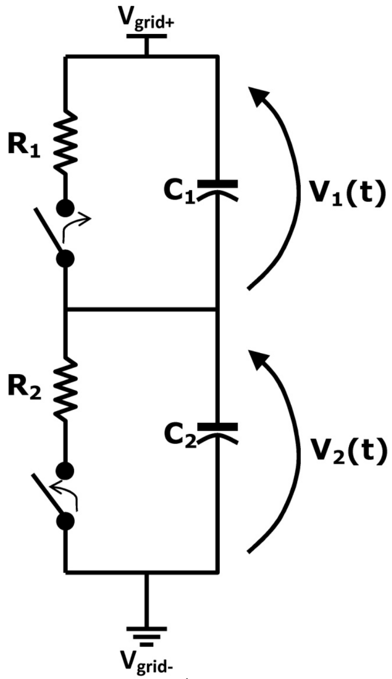
- Steady state assumption
- A non–trivial solution exists

Effective Impedance at Maximum Switching Rate

- Based on delivered charge and energy dissipation for every cycle

$$Q = \int_0^T dt I(t)$$





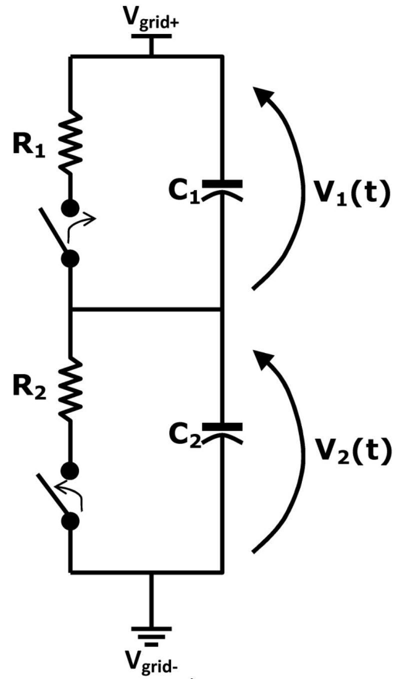
$$E_1 \equiv \exp\left[-\frac{T}{2R_1(C_1+C_2)}\right], \quad E_2 \equiv \exp\left[-\frac{T}{2R_2(C_1+C_2)}\right]$$

Effective Impedance at Maximum Switching Rate

- Based on delivered charge and energy dissipation for every cycle

$$Q = \int_0^T dt I(t)$$

$$C_0 = \frac{Q}{V_{CC}} = \left(C_1 + C_2\right)\frac{\left(1 - E_1\right)\left(1 - E_2\right)}{1 - E_1 E_2}$$



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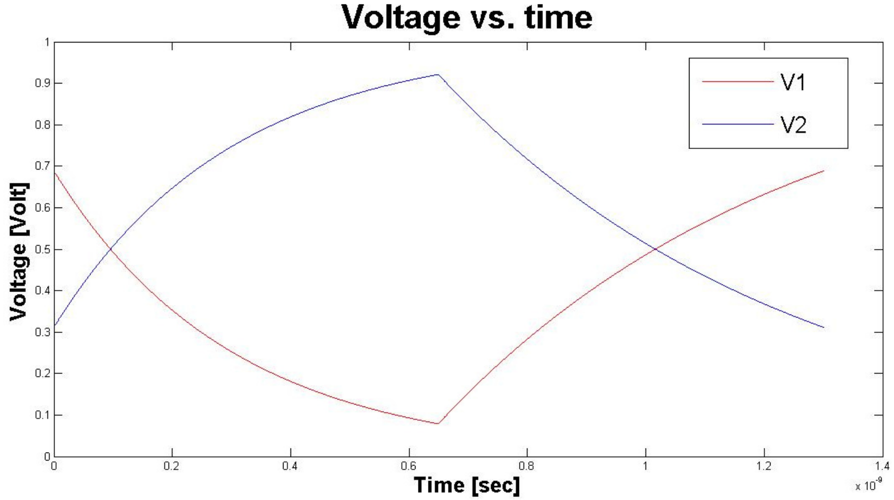
Effective Impedance at Maximum Switching Rate

- Based on delivered charge and energy dissipation for every cycle

$$W_D = \frac{1}{R_1}\int_0^{T/2} V_1^{\,2}(t)\,dt + \frac{1}{R_2}\int_{T/2}^{T} V_2^{\,2}(t)\,dt$$



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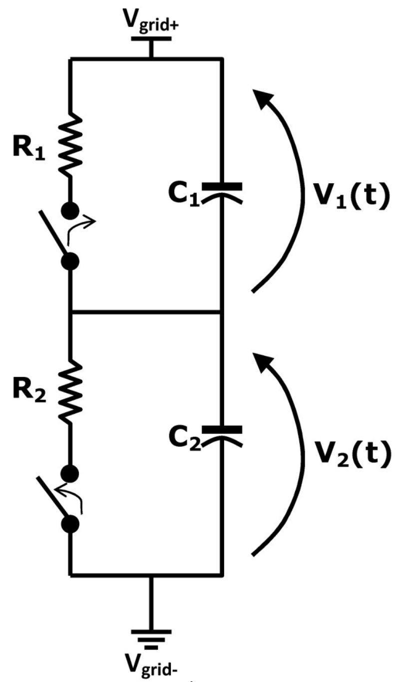

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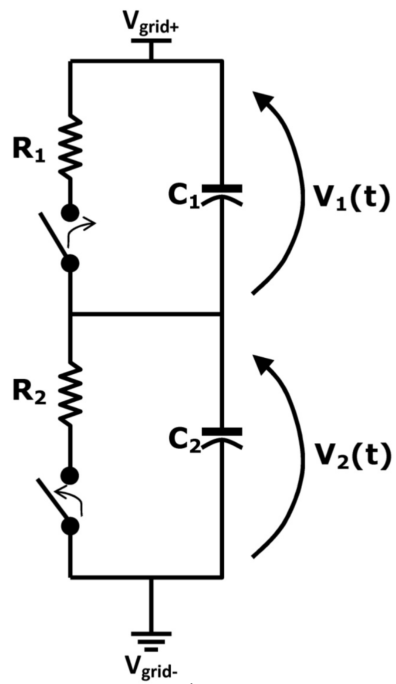
$$\mathrm{R}_0 = \frac{V_{CC}^2}{W_D}\cdot T = \frac{1-E_1E_2}{(C_1+C_2)(1-E_1)(1-E_2)}\cdot T$$



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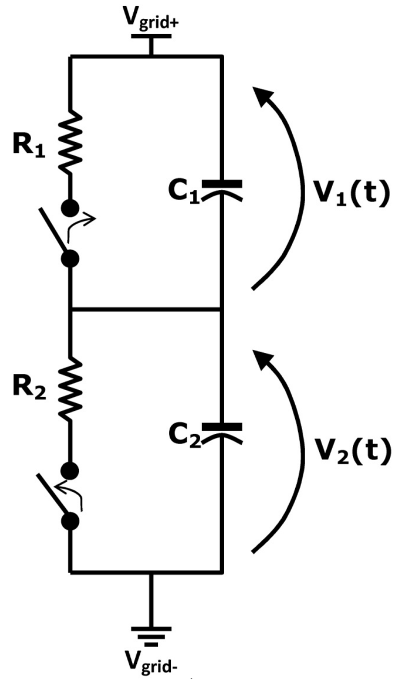
$$\mathrm{R}_0 = \frac{V_{CC}^{\;2}}{W_D}\cdot T = \frac{1-E_1E_2}{\left(C_1+C_2\right)\left(1-E_1\right)\left(1-E_2\right)}\cdot T = \frac{T}{C_0}$$



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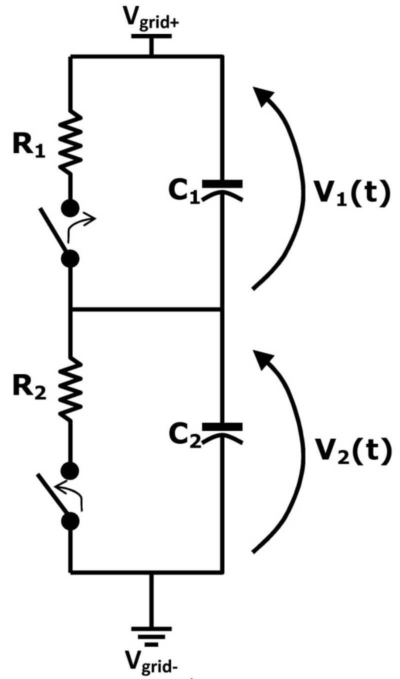
Effective Impedance at Maximum Switching Rate

- Based on delivered charge and energy dissipation for every cycle

$$C_0 = \frac{Q}{V_{CC}} = \left(C_1 + C_2\right)\cancel{\frac{\left(1-E_1\right)\left(1-E_2\right)}{1-E_1E_2}} = C_1 + C_2$$

$$\mathrm{R}_0 = \frac{V_{CC}{}^2}{W_D}\cdot T = \frac{1-E_1E_2}{\left(C_1+C_2\right)\left(1-E_1\right)\left(1-E_2\right)}\cdot T = \frac{T}{C_0}$$



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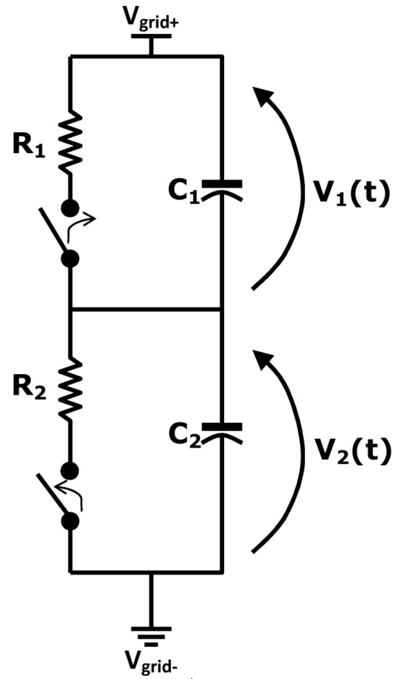
Effective Impedance at Maximum Switching Rate

- Based on delivered charge and energy dissipation for every cycle

$$\boxed{R_0 C_0 = T}$$

$$\boxed{C_0 = C_1 + C_2}$$



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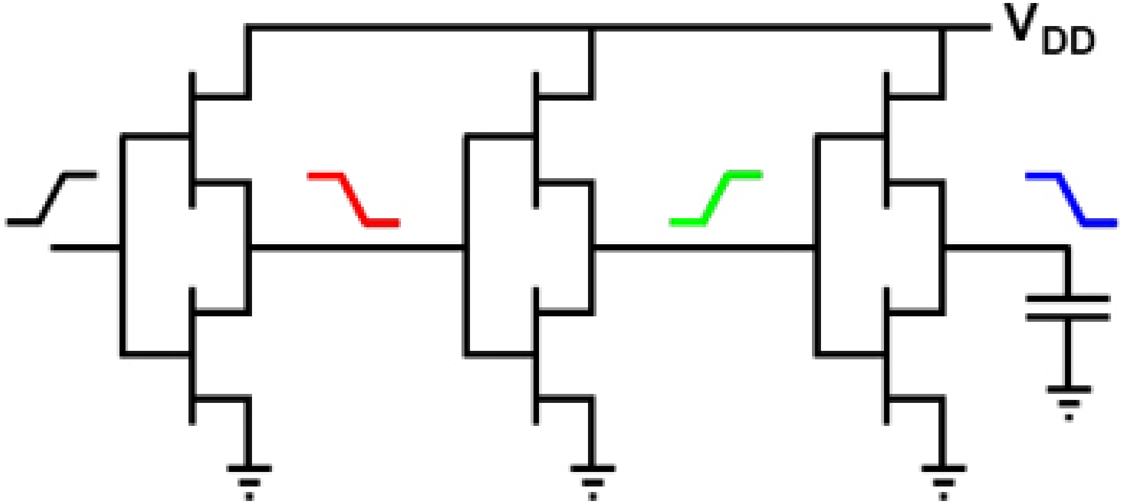
Macrocircuit

- Expandable to numerous parallel microcircuits



- For N microcircuits in parallel (N~10[6])

$$R_0 = R_{0,1} \parallel R_{0,2} \parallel \ldots \parallel R_{0,N}$$

$$C_0 = C_{0,1} + C_{0,2} + \ldots + C_{0,N}$$

Macrocircuit – Effective Impedance

- Actual switching rate is less than the maximum rate
- Activity function of the macro circuit is considered

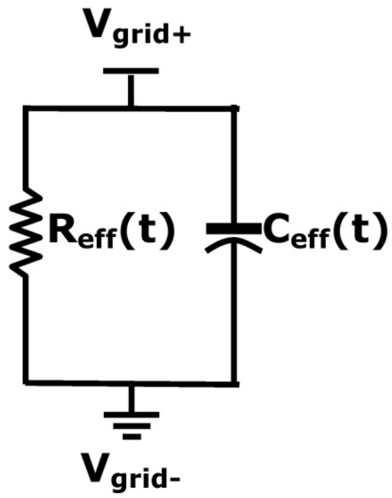
$$C_{eff}(t) = C_0 \cdot \alpha(t), \quad R_{eff}(t) = R_0 \cdot \frac{1}{\alpha(t)}$$

1nF

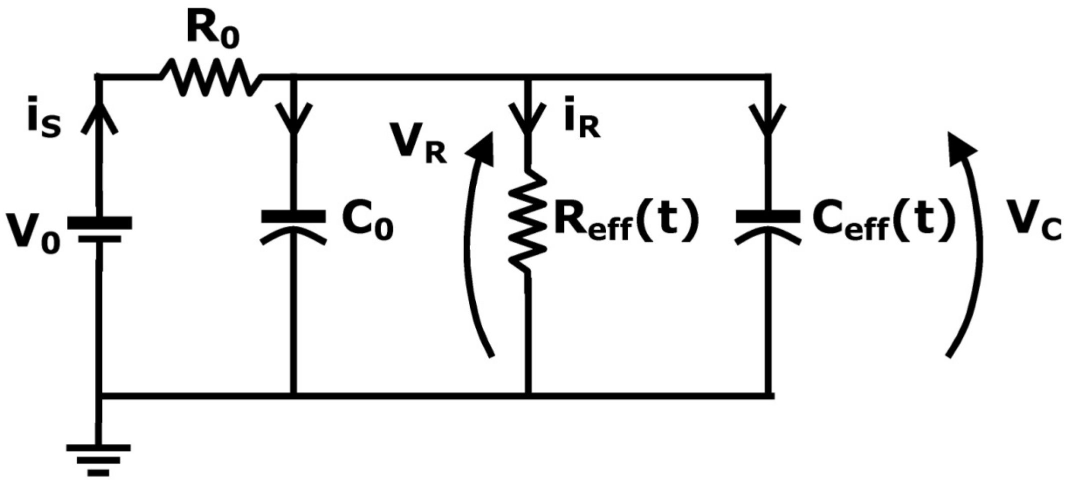
msec

10-100Ω

msec



Power Grid Analysis



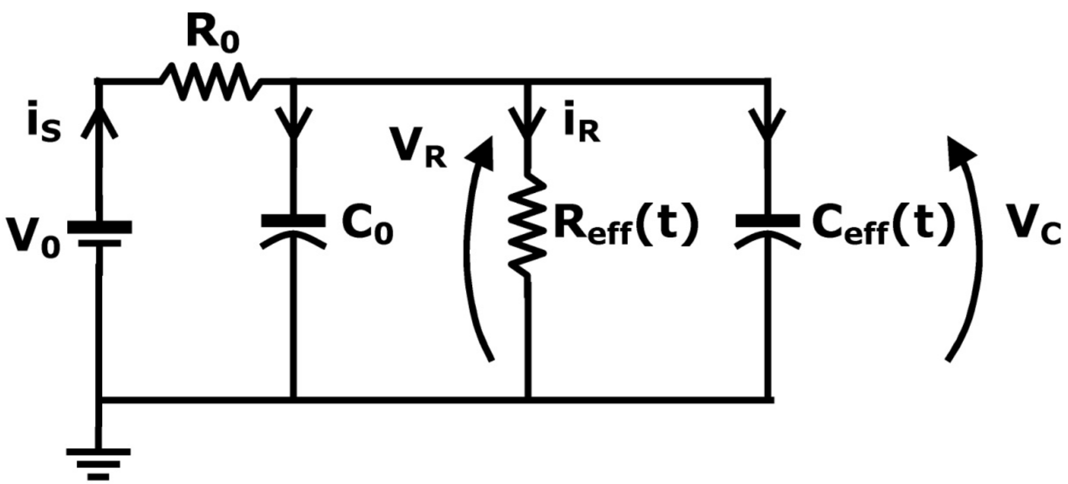
$$\tilde{C} = C_0 + C_{eff}(t)$$

$$\frac{1}{\tau(t)} \equiv \frac{1}{\tilde{C}(t)}\left[\frac{1}{R_{eff}(t)} + \frac{1}{R_0}\right]$$

$$Q_{\tilde{c}}(t) \equiv \tilde{C}(t) V_R(t)$$

$$\boxed{\frac{dQ_{\tilde{c}}(t)}{dt} + \frac{1}{\tau(t)} Q_{\tilde{c}}(t) = \frac{V_0}{R_0}}$$

Power Grid Analysis



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$$Q_{\tilde{c}}(t) \equiv \tilde{C}(t) V_R(t)$$

$$\boxed{Q_{\tilde{c}}(t) = Q_{\tilde{c}}(0) e^{-\int_0^t dt' \frac{1}{\tau(t')}} + \frac{V_0}{R_0} e^{-\int_0^t dt' \frac{1}{\tau(t')}} \cdot \left[\int_0^t dt' e^{\int_0^{t'} dt'' \frac{1}{\tau(t'')}}\right]}$$

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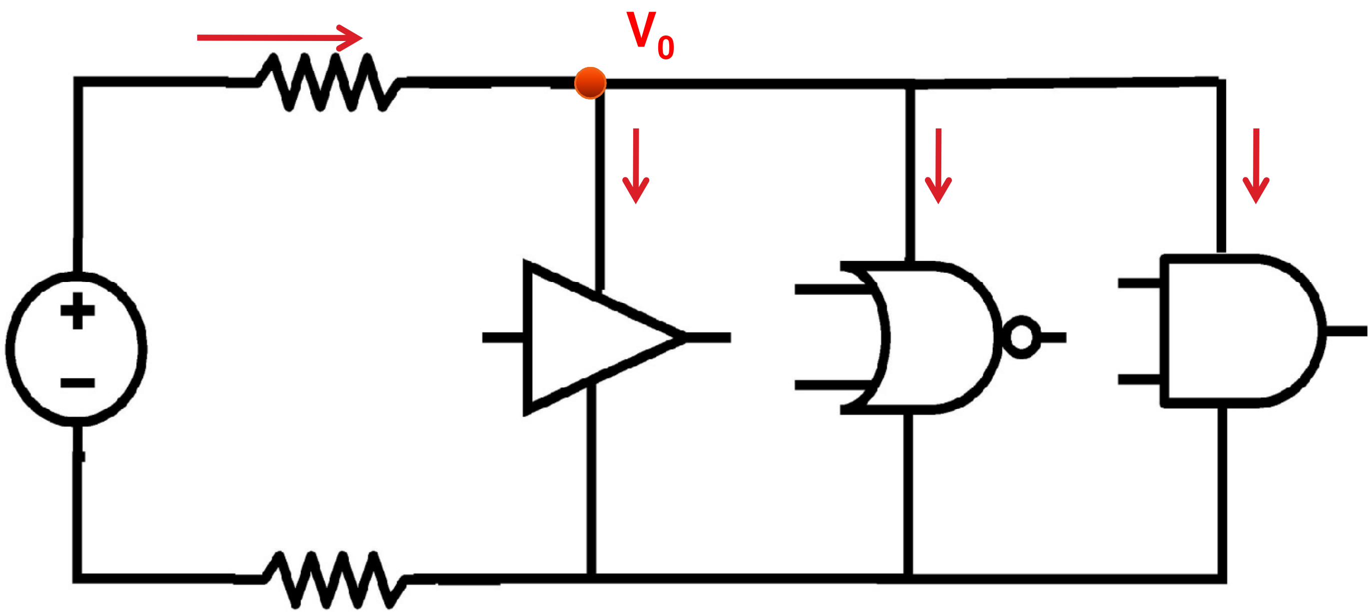
Feedback Between the Power Grid and Current Consumer

Negative feedback – decrease voltage drop



Ideal current source model

ignores this interaction

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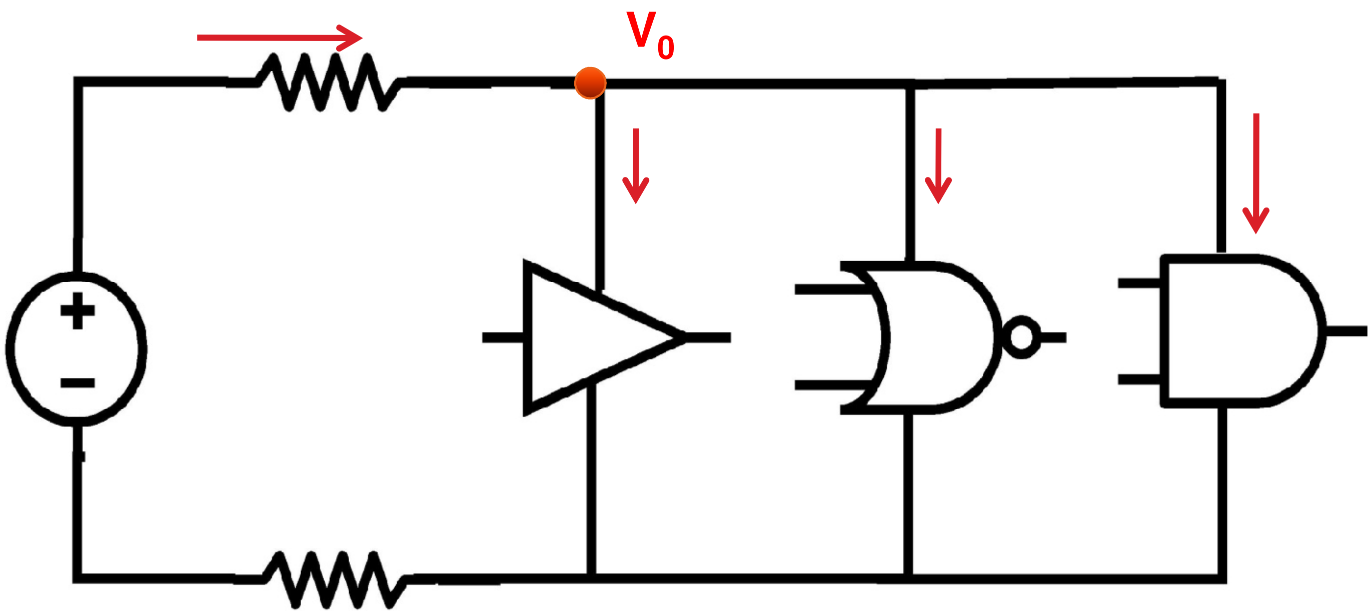

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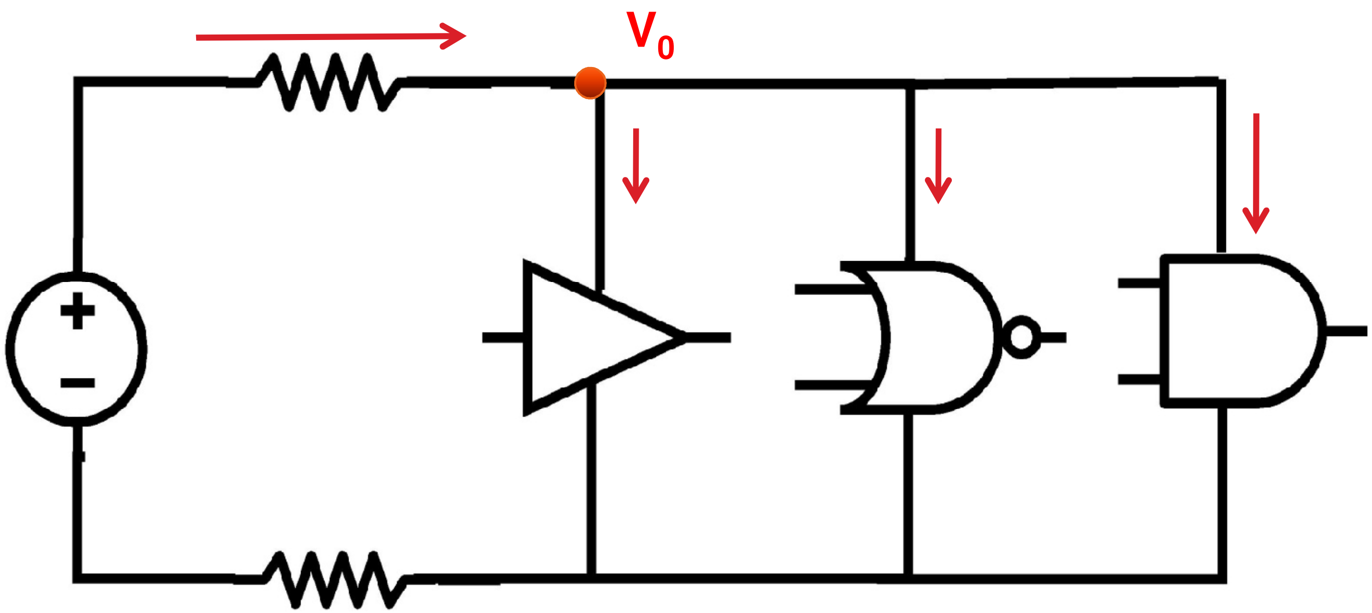

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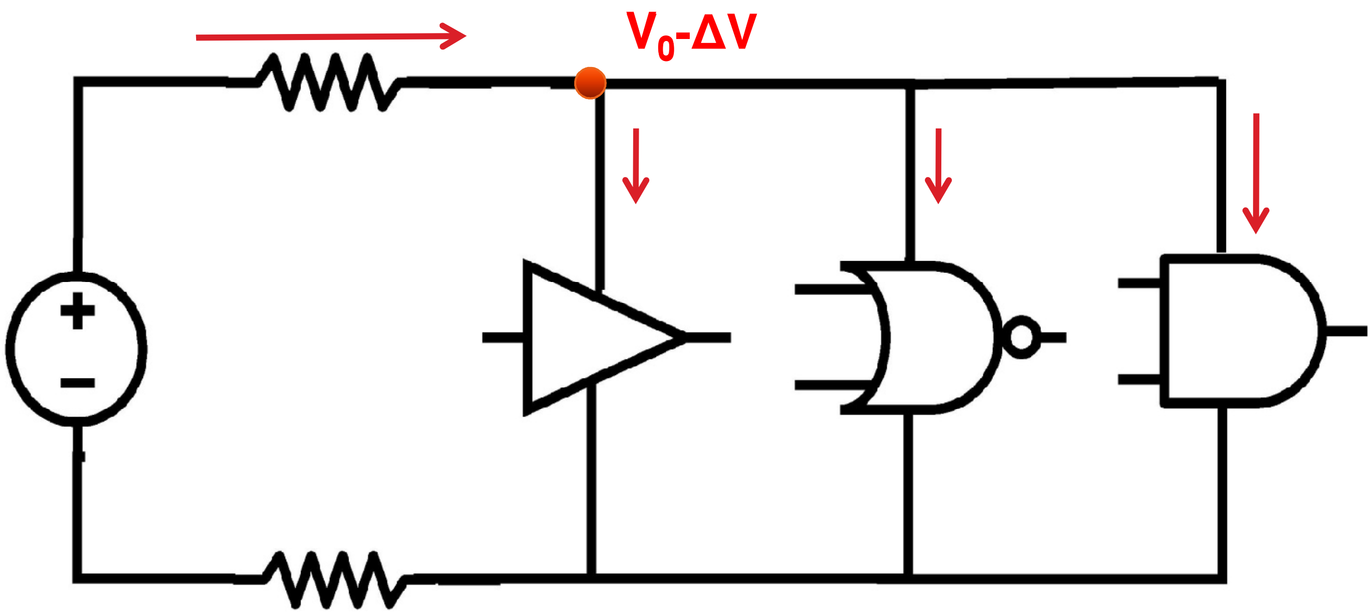

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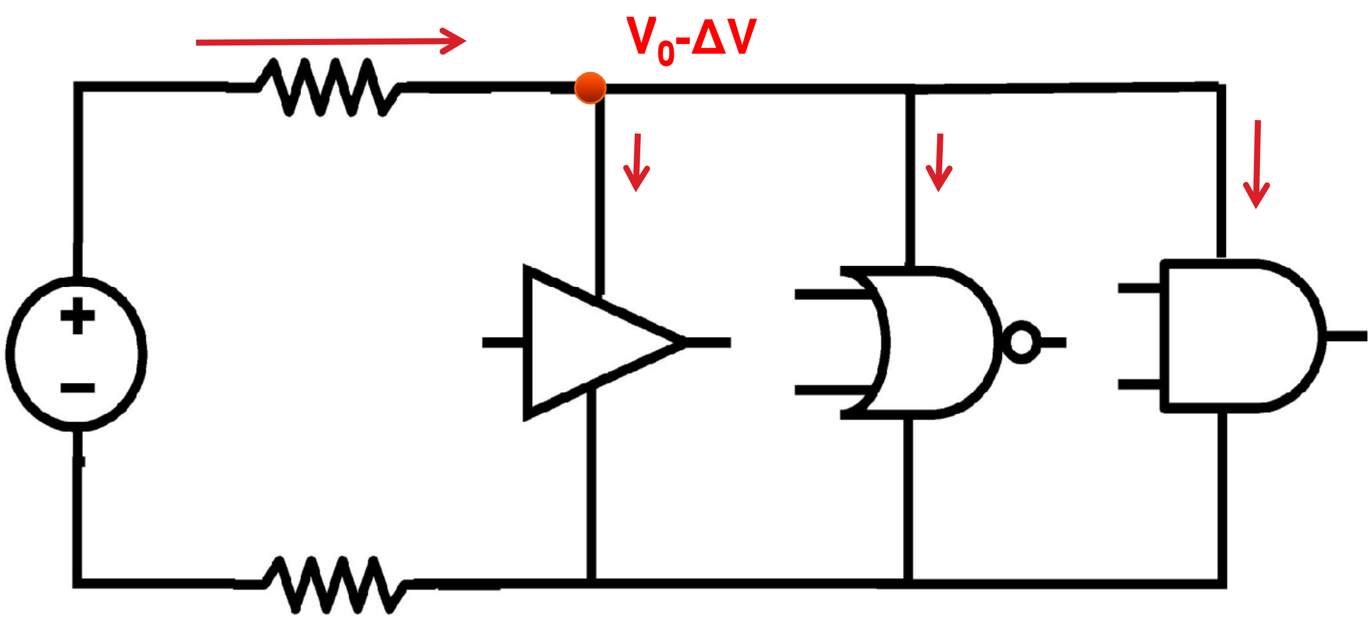

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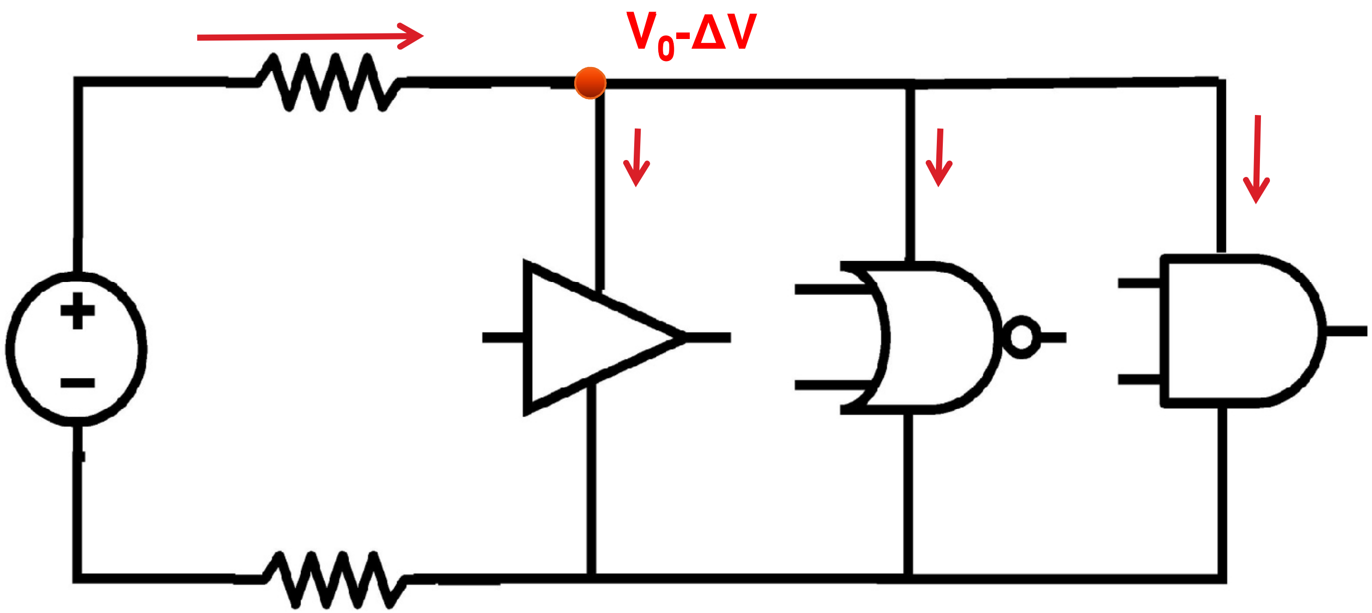

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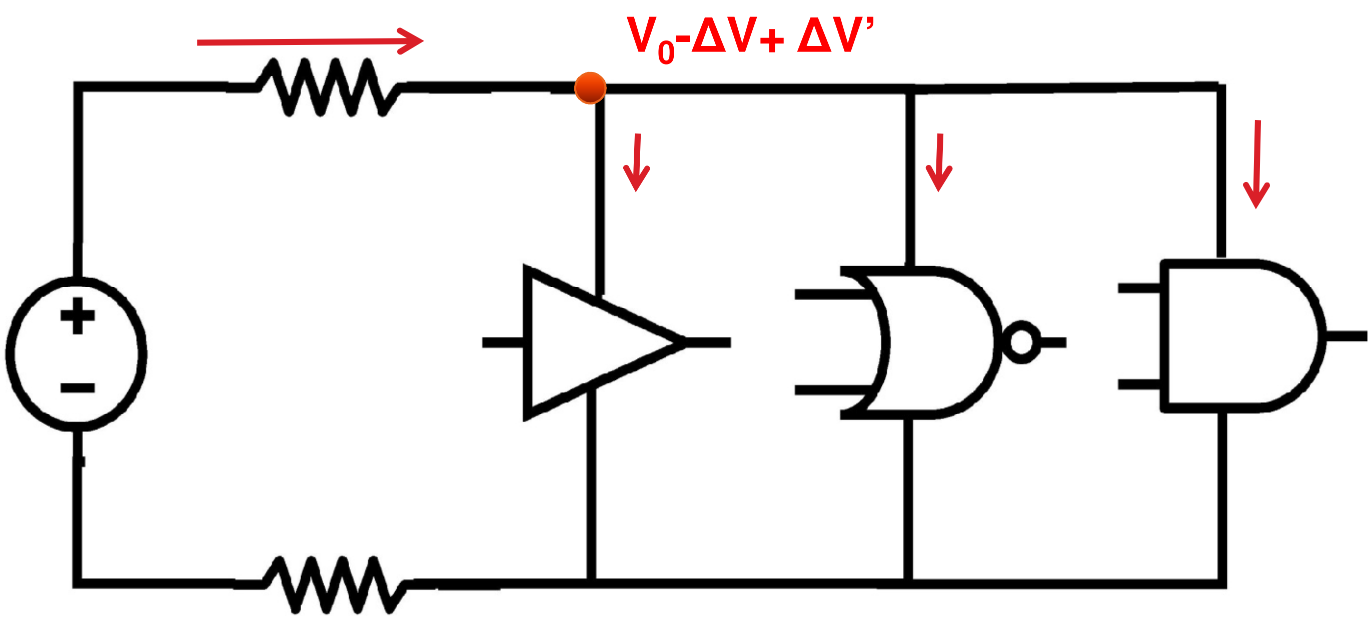
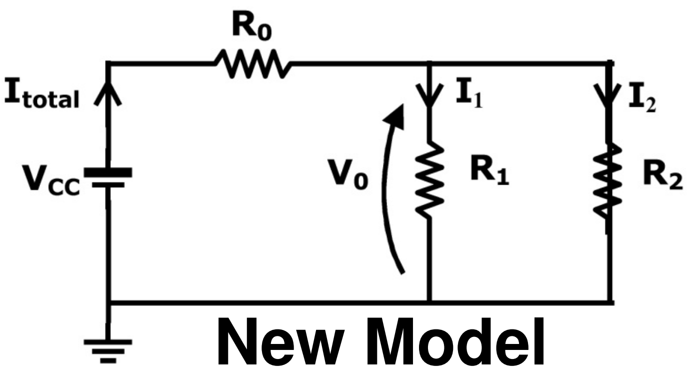
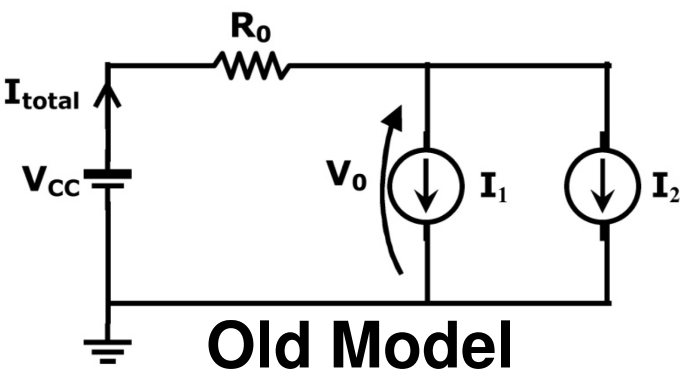
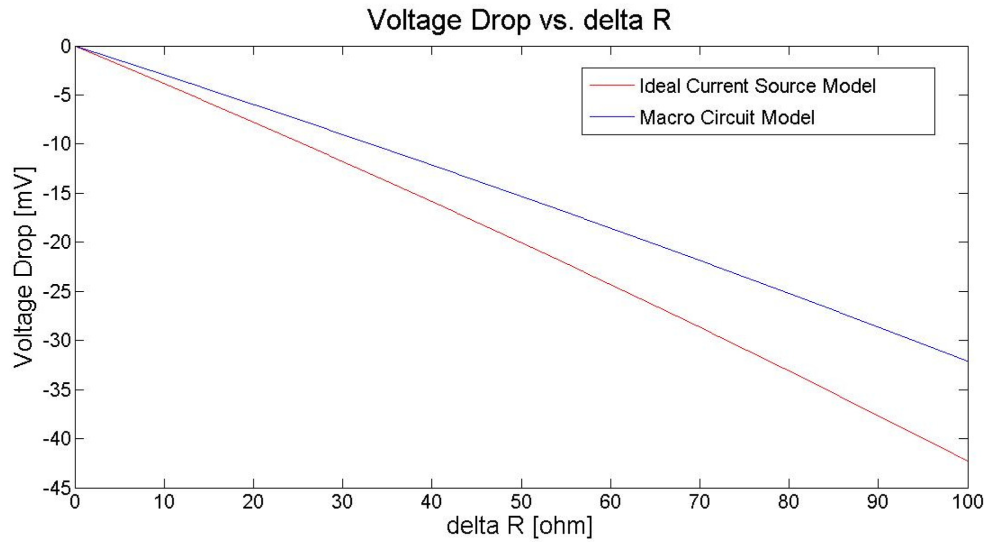


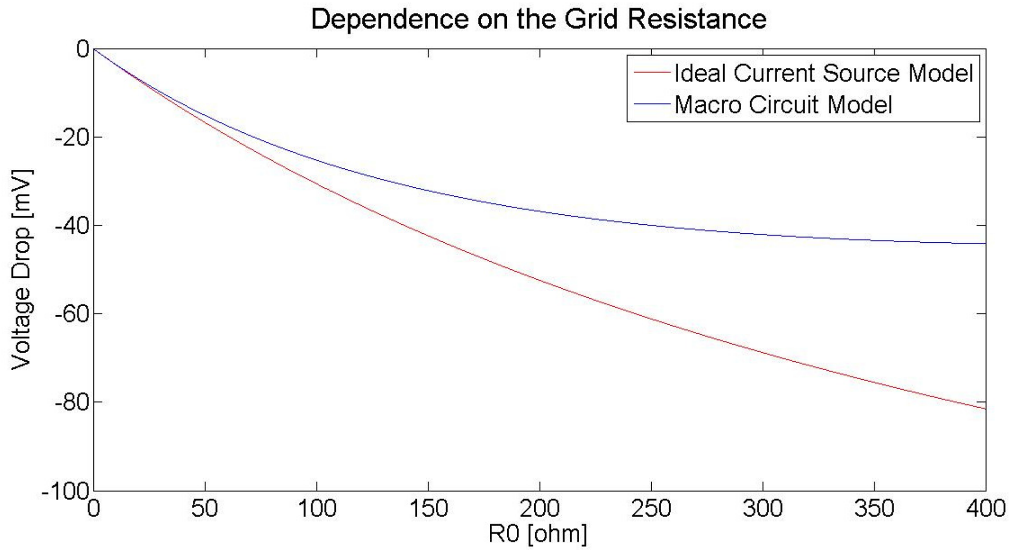
Ideal current source model ignores this interaction

Comparison Between the Models – Feedback Issue









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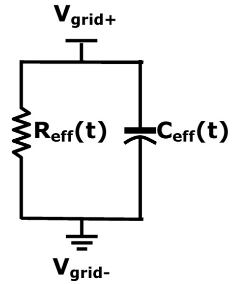
Summary



Old Model	New Model
Intuitive	Less intuitive
Know every node's voltage	Can't know every node voltage drop
Significant computational time	More time efficient than the conventional model
Active supply as a consumer – misleading	Based only passive elements
Ignores effect of voltage drop on the current consumer	Considers the interactions between the power grid and the element
Complex	Complex reduced

Thank You!